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STATIONARY DYNAMIC PROGRAMMING IN THE CRYPTOGRAPHIC CURRENCY RESEARCH

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Abstract: *The article is dedicated to the issue of applying a money search theoretical Kiyotaki's & Wright's (1993) model and its application by Hendrickson, Hogan & Luther (2014) to the cryptographic currency field. The main method for the analysis is a numerical implementation of the model, which allows to understand the model better and dwell on its adequacy. Although cryptomoney has been closely followed by an Internet-community, the main focus essentially belongs to financial engineering, market position, legal status and political economy of cryptographic money. In current research an emphasis is made on the quantitative macroeconomic side of the cryptocurrency and its co-existence with fiat money.*

Keywords: *cryptocurrency, money search model, Bellman equation, value function iteration, Bitcoin, virtual currency.*

INTRODUCTION

At present, opponents of fiat currencies discuss virtual currency and payment project Bitcoin as a practical form of critique of the current monetary and payment system (Weber, 2014). The political economy of Bitcoin and modelling co-existence of cryptomoney with legal tender money have become an interesting research topic. Cryptographic currency is a virtual or digital currency in form of a computer file that is highly encrypted for security reasons (Miller, 2015). Bitcoin occupies the leading position among over thousands of cryptocurrencies (viz. ¾ of this market).

Since it might be difficult to follow the model's description for an unprepared reader, it is worth mentioning that solving dynamical problems in macroeconomics is advanced field of researches. In general, there is a function with parameters, state and jump variables; jump variables are being unknowns, which need to be found to solve the equation. The function is called often a value function or Bellman equation. Elements of a value function have an economic meaning. But Bellman equation problem is that the unknown jump variables are spread over time; thus, it is impossible to find a closed-form solution. Therefore, different numerical methods are used to approximate the jump variables value, such that the equality of the value function holds. E.g., a value function iteration on a grid has been used to find numerical solution of the model.

MONEY SEARCH MODEL

Basically, the search dual currency model (Hendrickson, Hogan & Luther, 2014) has been developed on the basis of classic Kiyotaki's & Wright's search model (1993), who proved the possibility of equilibria with multiple currencies. The authors have constructed the model with a fiat currency and Bitcoin, where they have analyzed the equilibrium states and conditions for the cryptocurrency blocking by the state. The economic and technical cryptocurrency features have been analyzed in our previous research (Boiko, 2018).

The economy is populated by a set of agents A = [0,1], which is divided into G types. The number of goods equals the number of types G. These goods are indivisible and come in instorable units of size one. Each type of agents consumes a subset of goods of the same quantity denoted by n, but not quality, i.e. the consumption goods subset varies by type. A good belonging to such a subset is referred to as a consumption good. Consuming one of the consumption goods generates utility U. The production cost for each agent is C. Agents cannot consume their own output. The model assumes two types of indivisible, storable currencies: money and Bitcoin. The storage cost of money is δ_m and the storage cost of Bitcoin is δ_b. At the beginning of time, a fraction of agents M is endowed with one unit of money per agent; a fraction of agents B is endowed with one unit of Bitcoin and $1-M-B$ agents receive no endowment. Because of normalization of currencies to 1, $M \in (0.1)$ and $B \in (0.1)$, M also represents a total supply of money and B represents a total supply of Bitcoin. Since an individual must consume before producing, there will trade either through barter, money or Bitcoin.

It is worthy to note that a matching rule is not random like in the original paper (Kiyotaki's & Wright's, 1993), but agents choose deliberately with whom to trade. In the first stage, there is a non-random deliberate pairwise matching process and there is a random trading process on a second stage. The probability that a given agent wants to consume at the shop is $\rho = 1/n$. The authors compare it with situation, where agents, who wish to consume arrive at a place where they would like to buy, but might decide not to purchase (Hendricksen, Hogan & Luther, 2014). On the other hand, the agent in a shop has a choice whether to accept money, Bitcoin or consumption goods. This decision determines currencies' demand introduced via a probability of a random agent accepting money π and Bitcoin θ. These probabilities depend on solving a double-coincidence problem. Finally, Π and Θ are the best responses of the goods holder on whether to accept money and Bitcoin.

In the search model, agents demand money as an exchange medium. When highly acceptable, money can facilitate trade by increasing the number of matches between trading parties. In a barter economy, an exchange is possible only if an agent holding good i and willing to consume good j (ij agent) meets an individual holding good j willing to consume good i (ji agent). Money trade could occur in two steps. First, if agent ij trades with an agent mi (i.e. someone holding money m and willing to consume good i), he thereby becomes mj agent. Second, mj agent trades with ik agent (i.e. someone holding good i and willing to accept money m in order to trade for good k), thereby becoming an jj agent. Finally, jj agent consumes good j and derives utility. Lastly, one more probability variable needs to be introduced. It would correspond to Poisson arrival rate β, which measures probability of a pairwise meeting of agents in the original environment (Kiyotaki & Wright, 1993). Instead, the probability that monetary exchange takes place is used in a directed matching setting of this model.

For a money holder it is represented by a_m, for a Bitcoin holder by a_b. In addition, since a goods holder may decide between money and Bitcoin, he can experience both money monetary exchange with probability $a_{0,m}$ and Bitcoin monetary exchange with probability $a_{0,b}$. The probabilities of monetary exchange for money, Bitcoin and goods holders depend on the respective fractions of these agents M, B and $1-M-B$, which is shown as follows.

$$a_m = min\left\{1, \frac{1-M-B}{M}\right\}$$

$$a_b = min\{1, \frac{1-M-B}{B}\}$$

$$a_{0,m} = min\{1, \frac{M}{1-M-B}\}$$

$$a_{0,b} = min\{1, \frac{B}{1-M-B}\}$$

One more assumption is that once matched, each agent receives a preference shock regarding the qualitative goods preference in his consumption set. This assumption turns out to be crucial for the probabilities to hold monetary exchange. *Figure 1* depicts a small example economy populated by 10 agents, who are divided into 5 types G, each type having the same goods preferences.

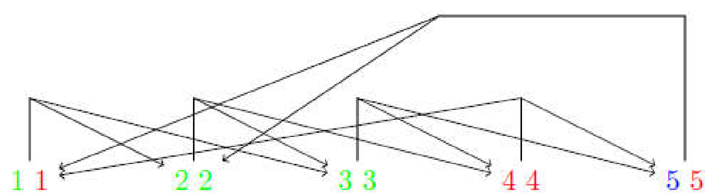

Figure 1: Endowment and preferences before the preference shock
Source: created by author

The number of consumption goods of each type is 2. Preferences are shown with arrows. 40% of the agents are endowment with money (shown in red), 10% with Bitcoin (blue) and 50% hold the goods (green). Directed matching means that preference arrows point to the right agents to go to. For instance, agent type-2 will go directly to agent-types 3 and 4. However, these preference arrows are not constant and subject to a preference shock after each match. The actual decision to buy is random. That is why the allocation of money, Bitcoin and goods is not depicted in color (*Figure 2*).

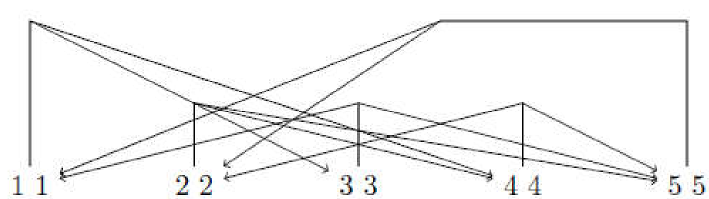

Figure 2: Preferences after the preference shock
Source: created by author

Formally, $G = 5$, $n = 2$ and the probability that any given agent wants to consume once bilaterally matched is $\rho = 1/n = ½$. By looking at currency supply $M = 0.4$, $B = 0.1$ and $1-M-B = 0.5$, we can determine probabilities of monetary exchange according to abovementioned formulas, i.e. $a_m = 1$, $a_b = 1$, $a_{0,m} = 0.8$ and $a_{0,b} = 0.2$. Since there is fewer money and Bitcoin holders than goods holders, the probability of a goods holder to make a money exchange $a_{0,m}$ or enter a Bitcoin monetary exchange $a_{0,b}$ is less than 1. Before looking at value functions of each state, remember that π is the probability that a random agent in the economy accepts money and θ is the probability that a random agent in the economy accepts Bitcoin. Moreover, $\Pi(\pi)$ and $\tau(\Theta)$ are best responses of a goods holder on whether to accept money and Bitcoin. The following are Bellman's Equations that are the value function equations for goods holder V_0, money holder V_m and Bitcoin holder V_b:

$$rV_0 = \left(1 - a_{0,m} - a_{0,b}\right)\rho^2(U - C) + \max_{\Pi\in[0,1]} a_{0,m}\Pi(\pi)\rho(V_m - V_0 - C) +$$
$$+ \max_{\Theta\in[0,1]} a_{0,b}\Theta(\theta)\rho(V_b - V_0 - C);$$
$$rV_m = a_m\pi\rho(U + V_0 - V_m) - \delta_m;\ rV_b = a_b\theta\rho(U + V_0 - V_b) - \delta_b.$$

Table 1 summarizes different variables from the search model.

Table 1

Parameters and variables of the dual currency search model

Symbols	*Definitions*
V_0	value function of a goods holder
V_m	value function of a money holder
V_b	value function of a Bitcoin holder
r	discount rate
$a_{0,m}$	probability that a goods holder is matched with a currency holder
$a_{0,b}$	probability that a goods holder is matched with a Bitcoin holder
ρ	probability that any given agent wants to consume a good
U	utility generated by consumption
C	production cost
π	1) probability that a random agent in the economy accepts money 2) fraction of agents willing to accept money
Π(π)	best response of a goods holder on whether to accept money
θ	1) probability that a random agent in the economy accepts Bitcoin 2) fraction of agents willing to accept Bitcoin
Θ(θ)	best response of a goods holder on whether to accept Bitcoin

Source: summarized by author

The gain of accepting money for a goods holder is represented by V_m-V_0-C and when weighted with probability $a_{0,m}\rho$ it becomes an expected value of accepting money for a goods holder. Gain of accepting Bitcoin is V_b-V_0-C becomes an expected value of accepting Bitcoin when weighted with the probability $a_{0,b}\rho(V_b - V_0 - C)$. If there is a gain of accepting money, i.e. $V_m-V_0-C > 0$, then each goods trader will accept it. If there is no gain of accepting money, that is $V_m-V_0-C = 0$, then goods traders are indifferent of accepting. If the gain of becoming a money holder is negative, a goods trader will never decide to do. The same holds for accepting Bitcoin (see equations below).

$$\pi = \Pi = \begin{cases} 0 & if\ V_m - V_1 - C < 0; \\ 1 & if\ V_m - V_1 - C > 0; \\ [0;1] & if\ V_m - V_1 - C = 0. \end{cases}$$

$$\theta = \Theta = \begin{cases} 0 & if\ V_m - V_1 - C < 0; \\ 1 & if\ V_m - V_1 - C > 0; \\ [0;1] & if\ V_m - V_1 - C = 0. \end{cases}$$

Steady states are primarily determined by the gain of accepting money $V_m - V_0 - C$ and Bitcoin $V_b - V_0 - C$. In symmetric equilibrium, the best response correspondence accounts for the path to the steady state. It implies that $\Pi(\pi)=\pi$ and $\tau(\theta)=\theta$. That is in equilibrium the probability that a random agent in the economy accepts money π equals to the best response of a goods holder of whether to accept money Π. Thus, the probability that a random agent in the economy accepts Bitcoin θ equals to the best response of a goods holder of whether to accept Bitcoin τ. Since population is normalized to 1, π may also be viewed as a fraction of agents willing to accept money and θ is a fraction of agents willing to accept Bitcoin. The system achieves equilibrium in a "one-step" procedure shown in *Figure 3*.

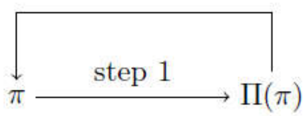

Figure 3: Path of π to the steady state

Source: created by author

When the system is out of the stationary state, a goods holder takes in consideration π, forms its best response Π, which than becomes π because all agents are supposed to form identical best responses. The same is for Θ, which will equal τ in equilibrium. As it is suggested by Kiyotaki & Wright (1993), the set of equilibria for Π is the set of fixed points of the best-response correspondence. In original paper with only one currency, the set comprises 3 elements. But in the model extended with cryptocurrency both set of equilibrium values for Π and τ need to be taken into account (*Figure 4*).

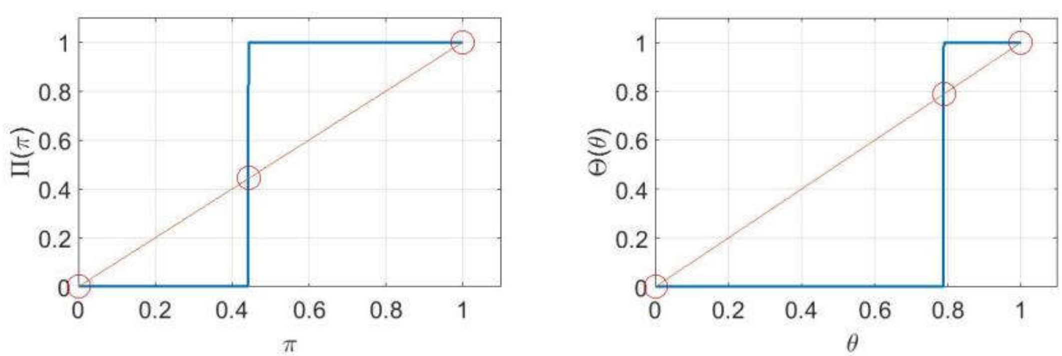

Figure 4: Best response correspondence for money and Bitcoin (equilibria are marked with red circles)

Source: created by author

Critical values in Figure 4 are $\pi = 0.443$ and $\theta = 0.789$. The model implies that the relation of probability that a random agent in the economy accepts money to its critical value indicates the equilibrium of π value. The reason for this is that a threshold probability is probability of accepting currency when the gain for doing so is zero. When $\pi \geq \hat{\pi}$, then it becomes partially/fully accepted. When $\pi < \hat{\pi}$, then nobody accepts it. When $\theta \geq \hat{\theta}$, then Bitcoin becomes partially/fully accepted as well, but nobody will accept it in case $\theta < \hat{\theta}$. As it can be seen from the graphs, there are 9 possible equilibria, which are grouped into 4 groups: (1) money equilibrium, where money, but not Bitcoin, is accepted; (2) Bitcoin equilibrium, where Bitcoin, but not money, is accepted; (3) coexistence of money and Bitcoin, in which both money and Bitcoin are accepted in exchange; (4) non-monetary equilibrium where neither currency is accepted (Hendrickson, Hogan & Luther, 2014).

Determining threshold values is important for the equilibrium. According to Hendrickson et all, the unique value of $\pi = \hat{\pi}$ that sets $V_m - V_0 - C = 0$ is then a threshold value for accepting money. Equivalently, $rV_m - rV_0 - rC = 0$ combined with Bellman's Equations yields (see formula below).

$$\hat{\pi} = \begin{cases} \dfrac{(1 - a_{0,m} - a_{0,b})\rho}{a_m} + \dfrac{\delta_m + rC}{a_m \rho (U - C)} + \dfrac{a_{0,b}(V_b - V_0 - C)}{a_m (U - C)} & if \theta = 1; \\ \dfrac{(1 - a_{0,m} - a_{0,b})\rho}{a_m} + \dfrac{\delta_m + rC}{a_m \rho (U - C)} & if\ \theta < 1. \end{cases}$$

The core insight here is that a critical value for accepting or not accepting money $\pi = \hat{\pi}$ depends on whether Bitcoin is commonly accepted. It shows the competing nature of money and Bitcoin. When $\theta = 1$, the critical value $\hat{\pi}$ is higher. It means that equilibrium value for $\pi = 0$ is more likely than the other two equilibrium values. On the contrary, when $\theta < 1$, the critical value $\hat{\pi}$ is smaller. It means higher chances that in out-of-equilibrium state $\pi > \hat{\pi}$ and the resulting equilibrium is complete acceptance of money by all agents.

Combining $V_b - V_0 - C = 0$ with Bellman's Equations for the threshold probability of a random agent accepting Bitcoin $\hat{\Theta}$ yields (see formula below).

$$\hat{\theta} = \begin{cases} \dfrac{(1 - a_{0,m} - a_{0,b})\rho}{a_b} + \dfrac{\delta_b + rC}{a_b \rho (U - C)} + \dfrac{a_{0,m}(V_m - V_0 - C)}{a_b (U - C)} & if \theta = 1; \\ \dfrac{(1 - a_{0,m} - a_{0,b})\rho}{a_b} + \dfrac{\delta_b + rC}{a_b \rho (U - C)} & if\ \theta < 1. \end{cases}$$

It is very unlikely that a system can settle at interim equilibria of partial acceptance probabilities $\pi = \hat{\pi}$ and $\theta = \hat{\theta}$. Partial acceptance equilibrium requires that gain from holding currency is zero as well as the fraction of agents willing to accept that currency exactly equals threshold probability.

CONCLUSIONS

Presented model of exchange captures core features inherent for cryptographic money. It addresses double coincidence of wants problem, bilateral and directed nature of exchange in digital environment and predominance of generally accepted fiat money. The competing position of cryptocurrency Bitcoin according to the fiat money is reflected in that the acceptance of one currency makes more difficult to be also accepted for the other. Although the model shows 9 possible equilibria, we have to note that partial acceptance equilibrium seems very unlikely for any currency. Indeed, a fraction of agents willing to accept each currency is important for the acceptance decisions in equilibrium.

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